Chapter 12: Direct restrictions on variables

Steps optimization without restrictions:

1. Compute the first order conditions (FOCs) f_1 and $f_{2.}$

2. Solve FOCs

3. Compute the **Hessian Matrix (H):**

$$\begin{bmatrix} f_{11} & f_{12} \\ f_{21} & f_{22} \end{bmatrix}$$

4. $H_1 = f_{11}$ and H_2 = determinant of H

    - if H_1(abs) < 0 and H_2 (abs) > o → the point is a maximum

    - if H_1(abs) > 0 and H_2 (abs) > o → the point is a minimum

    - if f_{11} and f_{22} have opposite signs → the point is a saddle point

Steps optimization with restrictions:

1. Compute the first order conditions (FOCs) f_1 and f_2. From these conditions one can calculate x_1 and x_2.

2. Check for an interior solution. If x_1 and x_2 are both in the given range, then there is an interior solution. Then set up the **Hessian Matrix (H):**

$$\begin{bmatrix} f_{11} & f_{12} \\ f_{21} & f_{22} \end{bmatrix}$$

   $H_1 = f_{11}$ and H_2 = determinant of H

   - if H_1(abs) < 0 and H_2 (abs) > o → the point is a maximum
   - if H_1(abs) > 0 and H_2 (abs) > o → the point is a minimum
   - if f_{11} and f_{22} have opposite signs → the point is a saddle point
   - otherwise, the point is neither an extremum or saddle point.

3. Check for boundary solutions

4. Derive the boundary solutions

5. Check theorem: condition for maximum or minimum

One of the following conditions must hold for a **maximum**:

- $f_i(x^*) \leq 0$ and $(x^*_i - a_i)\, f_i(x^*) = 0$
- $f_i(x^*) \geq 0$ and $(b_i - x^*_i)\, f_i(x^*) = 0$

for all i = 1, …, n

One of the following conditions must hold for a **minimum**:

- $f_i(x^*) \geq 0$ and $(x^*_i - a_i)\, f_i(x^*) = 0$

- $f_i(x^*) \leq 0$ and $(b_i - x^*_i)\, f_i(x^*) = 0$

for all $i = 1, \ldots, n$

Note: if $a_i < x^*_i < b_i$ then both conditions hold.