Chapter 12: Equity Risk Premium

Expected return of a Portfolio

A portfolio can be described by its portfolio weights: the fraction of the total investment in the portfolio held in each individual investment in de portfolio.

$$Wi = \frac{Value\ of\ investment\ i}{Total\ value\ portfolio}$$

The return on a portfolio (Rp) is the weighted average of the returns on the investments in the portfolio, where the weights correspond to the portfolio weights.

Rp = w1 R1 + w2 R2 + … + wn Rn

The expected return of a portfolio is the return you can expect to earn on your portfolio, it is the weighted average of expected returns of the investments in a portfolio, where the weights correspond to the portfolio weights.

E [Rp] = w1 E [R1] + w2 E [R2] + … + wn E [Rn].

Volatility of portfolio

The volatility of a portfolio is the total risk, measured as a standard deviation, of a portfolio. Recall that risk is reduced through diversification by combining stocks into a portfolio. The degree to which the stocks face common risks and move together determines the amount of risk that is eliminated.

Correlation is a measure of the degree to which returns share common risk. It's a degree to which stocks' returns move together. Correlation is always between -1 and 1. A correlation of -1 means: returns move in opposite directions. A correlation of 0: uncorrelated. A correlation of 1: returns tend to move together.

$$Correlation = \frac{covariance\ of\ the\ returns}{standard\ deviation\ of\ each\ return}$$

$$Corr\ (Ri, Rj) = \frac{Cov\ (Ri, Rj)}{SD(Ri)SD(Rj)}$$

Now we can compute the portfolio variance:

$$Var\ (Rp) = W1^2SD(R1)^2 + W2^2SD(R2)^2 + 2W1W2Corr(R1, R2)SD(R1)SD(R2)$$

The first part accounts for the risk of stock 1, the second part for the risk of stock 2. The third part is the adjustment for how much the two stocks move together.

The benefits of diversification will rise when the amount of stocks in a portfolio increases. In a large portfolio, only risk that is common to all of the stocks (systematic risk) will matter.

An equally weighted portfolio is a portfolio in which the same dollar amount is invested in each stock.

Systematic risk

Only systematic risk is related to return, unsystematic isn't. However, standard deviation measures total risk (systematic and unsystematic risk). We need a way to measure just the systematic risk.

If all investors diversify their portfolios, only systematic risk will remain. If all investors do this optimally, the aggregate portfolio held by all investors is a fully-diversifies, optimal portfolio: market portfolio: the portfolio of all risky investments, held in proportion to their value. This portfolio contains all shares outstanding of every risky security.

Market capitalization is the total market value of equity.

Market capitalization = (number of shares outstanding) x (price per share).

A value-weighted portfolio is a portfolio in which each security is held in proportion to its market capitalization. Because a market portfolio only contains systematic risk, we can calculate the amount of systematic risk a stock has by looking at the sensitivity of a stock's return to the overall market.

Because it isn't possible to collect and update returns on all risky assets in the world, we use a market proxy: a portfolio whose return is believed to closely track the true market portfolio. Market indexes are the most common proxy portfolios. A market index is the market value of a broad-based portfolio of securities.

To calculate the amount of systematic risk, we use the relationship between individual stock's returns and the market portfolio's returns. A stock is highly sensitive to systematic risk, if the return of the stock is highly sensitive to the return of the market portfolio. A stock has got little systematic risk, if the returns of the stock do not depend on the market's returns.

To estimate the sensitivity of a stock to the market portfolio (the systematic risk), we use the stock's beta (β): the expected percent change in the excess return of a security for a 1% change in the excess return of the market portfolio. Beta represents the amount by which risks that affect the overall market are amplified or dampened in a given stock. A security has got a high beta if it tends to move more than the market, a security has got a low beta if it tends to move less than the market.

The beta is the Cov (Ri,Rmkt) divided by the Var (Rmkt). The beta of a risk-free investment is always zero, because this investment has no correlation and volatility with the market (the return is known in advance). The beta of a market portfolio is always 1.

The capital asset pricing model

The expected return on any investment is the sum of a risk-free rate of return, as a compensation for inflation and the time value of money, and a risk premium for systematic risk.

Expected return = risk-free rate + risk premium for systematic risk.

Expected return for investment i = risk-free rate + βi x risk premium per unit of systematic risk.

The risk premium per unit of systematic risk can be estimated with the market risk premium (equity risk premium): the historical average excess returns on the market portfolio.

The *capital asset pricing model* (CAPM) is an equilibrium model of the relationship between risk and return that characterizes a security's expected return based on its beta with the market portfolio.

$$E[Ri] = rf + \beta i\,(E[Rmkt] - rf)$$

In this formula $\beta i\,(E[Rmkt] - rf)$ represents the risk premium for security i. This premium is multiplied by the beta. Investors will not invest unless they can expect the return E [Ri], for that reason this return is also called the required return: the expected return of an investment that is necessary to compensate for the risk of undertaking the investment.

Putting the outcomes in a graph: there is no relationship between the standard deviation (total risk) and the expected return of a stock. But there is a linear relationship between the beta and the expected return: the security market line (SML). This is the pricing implication of the CAPM, it specifies a linear relation between the risk premium of a security and its beta with the market portfolio. The SML applies to all securities, also to portfolios.

The beta of a portfolio is the weighted average beta of the securities in the portfolio:
$\beta p = w1\ \beta 1 + w2\ \beta 2 + \ldots + wn\ \beta n$