Chapter 13: Constrained optimization

In constrained optimization problems Lagrange multipliers are used. The Lagrangian **function**, with x_1 and x_2, is:

$L(x_1, x_2, \lambda) = f(x_1, x_2) + \lambda\, g(x_1, x_2)$

Steps for constrained optimization with equality constraints

Format of the problem is: max y = … s.t. $x_1 = a$

1. Identify the objective function f(x)
2. Rewrite the constraint as g(x) = 0
3. Construct the Lagrangian $L(x, \lambda) = f(x) + \lambda\, g(x)$
4. Compute the FOCs of the Lagrangian:
   - $dL/dx_1 = 0$, $dL/dx_2 = 0$ etc.
   - $dL/d\lambda = 0$
5. Solve FOCs

The **Bordered Hessian, H***, is:

$$H^* = \begin{matrix} f_{11} + \lambda g_{11} & f_{12} + \lambda g_{12} & g_1 \\ f_{21} + \lambda g_{21} & f_{22} + \lambda g_{22} & g_2 \\ g_1 & g_2 & 0 \end{matrix}$$

- If the determinant of H* > 0, then we have a maximum.
- If the determinant of H* < 0, then we have a minimum.

Weierstrass's Theorem: If f is a continuous function and x is a nonempty, closed and bounded set, then f has both a minimum and a maximum on x.

If in a constrained maximization problem f is quasi-concave and all g functions are quasi-convex, then any locally optimal solution is also globally optimal.