Chapter 16: Integration

The integral of a function is found by the process of anti-differentiation. This process is referred to as **integration**. Some rules regarding integration are:

- Power rule:

  $$\int x^n \, dx = \frac{x^{n+1}}{n+1} + C$$

- Integral of a sum

  $$\int [f(x) \pm g(x)] dx = \int f(x) dx \pm \int g(x) dx$$

- Integral of a constant multiple

  $$\int k f(x) dx = k \int f(x) dx$$

- Exponential rule

  $$\int e^x \, dx = e^x + C$$

- Logarithmic rule

  $$\int \frac{1}{x} dx = \ln(x) + C$$

The **Riemann integral,** also called definite integral, of a function defined on some interval is the area underneath the curve over that interval.

The **fundamental theorem of integral calculus:** if a function f(x) is continuous on the closed interval (a,b) and if F(x) is any anti-derivative of f(x), then:

$$\int_a^b f(x) dx = [F(x)]_a^b = F(b) - F(a)$$

The **producer surplus** can be calculated as:

$$PS = p_0Q_0 - \int_0^{q_0} MC(q)dq$$

It represents the change in profit.

The **consumer surplus** can be calculated as:

$$CS = \int_{p_0}^{p*} D(p)dp$$

Where p* stands for the chocke price; the price where demand, D, is equal to zero.

$$\int_a^b f(g(x))g'(x)\, dx = \int_{g(a)}^{g(b)} f(t)\, dt$$

The **average,μ,** can be calculated with the following formula:

$$\mu = \int_a^b x\, f(x)\, dx$$

σ^2 can be found with the formula:

$$\sigma^2 = \int_a^b (x-\mu)^2 f(x)\, dx = \int_a^b x^2 f(x)\, dx - \mu^2$$

Two rules to remember:

- if f'' > 0 → the function is convex and has a minimum
- if f'' < 0 → the function is concave and has a maximum

The fraction of students passing an exam can be calculated as:

Probability ($5.5 \leq x \leq 10$) = F(10) – F(5.5)

The average grade of those who pass an exam can be calculated as: