Chapter 23: Interest rates

Immunization is the organization of a portfolio of fixed income investments so that they will have a riskless value at some future date. *Convexity* is the calculation of the curvature in the price-yield relationship.

DV01 is a measure of how much bond's price will raise in response to a one basis point turn down in a bond's yield to maturity. If the yield and interest rates are the same, the DV01s will be useful for estimating interest rate risk.

High DV01 bonds are more sensitive to interest rate movements than low DV01 bonds, so they are also more volatile. If a corporation measures the risk exposure of its stock as having a negative DV01, acquiring a bond position with a positive DV01 will reduce the interest rate exposure of the corporation's equity.

Various ways to compute the DV01:

price at yield one basis point under the existing yield less price at existing yield.

use calculus to compute the derivative of the price-yield relationship directly: $DV01 = -.0001\frac{dP}{dr}$

r , the bond yield, is given in decimal form. This given a somewhat different answer than method 1 because a one basis point decline, is not infinite small.

price at yield ½ basis point below less price at ½ basis point above existing yield

One can use DV01 to estimate the price change of a bond or a bond portfolio for small changes in the level of interest rates, as measured by the bond's yield. The formula for this estimate:

$$\Delta P = -DV01 \times (\Delta bp)$$

10,000 Δ *r* is the yield change in number of basis points

ΔP = the change in the bond's price

Δbp = the interest rate change (in basis points)

Two rules when computing the DV01 of a portfolio of bonds:

- To change a DV01 per €100 face value to a DV01 per actual face value, multiply the actual face value of the portfolio's position in the bond by the DV01 per €100 face value and then divide by €100.

- To translate a DV01 per €1 million face value to a DV01 per actual face value, multiply the real face value of the portfolio's position in the bond by the DV01 per €1 million face value and then divide by €1 million.

If the term structure of interest rates is flat, a bond portfolio with a DV01 of zero has no sensitivity to interest rate movements.

Let r_n and r_m denote the annualized yield to maturity of the same bond computed with yields compounded *n* times a year and *m* times a year, respectively.

The DV01 for a bond (or portfolio) using compounding of *m* times a year is $\left(1+\frac{r_n}{n}\right)\Big/\left(1+\frac{r_m}{m}\right)$ times the DV01 of the bond using a compounding frequency of *n* times a year.

The *duration* of a bond, denoted DUR, is a weighted average of the waiting times (measured in years) for receiving its promised future cash flows.

The weight on each time is proportional to the discounted value of the cash flow to be paid at that time; that is, letting *r* denote the yield for the bond, P denote the bond's market price, and C_t denote the cash flow at date *t*,

the duration DUR of the bond is $$DUR = \frac{\left[\frac{C_1}{(1+r)}\right]1+\left[\frac{C_2}{(1+r)^2}\right]2+\ldots+\left[\frac{C_T}{(1+r)^T}\right]T}{\frac{C_1}{(1+r)}+\frac{C_2}{(1+r)^2}+\ldots+\frac{C_T}{(1+r)^T}}$$

$$= \sum_{t=1}^{T}\left[\frac{PV(C_t)}{P}\right]t$$

The weights, $PV(C_t)/P$ always sum to 1.

Premium bonds have lower durations, and discount bonds, being more like zero-coupon bonds, have higher durations. The period of a straight coupon bond with semi-annual payments increases at coupon dates.

Assuming that the term structure of interest rates is flat, the duration of a portfolio of bonds is the portfolio-weighted average of the durations of the respective bonds in the portfolio.

When the term structure of interest rates is flat, duration always can be viewed as minus the percentage change in the value of the bond with respect to changes in its continuously compounded yield to maturity. A bond maturing in *T* years, with cash flows of C_t at date *t*,

$t = 1,\ldots,T$, has a price of $P = \sum_{t=1}^{T} C_t e^{-rt}$

If *r* is the bond's continuously compounded yield to maturity.

The percentage change in P with respect to the continuously compounded yield is

$$\frac{dP/P}{dr} = -\frac{1}{P}\left(\sum_{t=1}^{T} tC_t e^{-rt}\right) = -DUR$$

Because DV01 can be translated into duration and vice versa, DV01 and duration are equivalent as tools both for measuring interest rate risk and for hedging.

If DV01 is based on rates that are compounded *m* times a year instead of continuously, the formula relating to DV01 to duration is modified as follows:

$$DV01 = \left[\frac{DUR}{1+\frac{r}{m}}\right] \times P \times .0001 = \textit{modified duration}$$

One can compute an *effective duration* of a corporate asset or liability with risk by first estimating its DV01 as an interest rate sensitivity in a factor model and then inverting the equation above to get DUR.

A perfect hedge makes the new portfolio have a DV01 of zero. To find this, the ratio of the duration of the unhedged bond portfolio (DUR_B) to the duration of its hedge portfolio (DUR_H) must be inversely proportional to the ratio of the respective market values of bonds;

that is $\dfrac{DUR_H}{DUR_B} = \dfrac{P_B}{P_H}$

If the term structure of interest rates is flat, immunization 'guarantees' a fixed value for an immunized portfolio at a horizon date. The value obtained is similar as the face value of a zero-coupon bond with (1) the same market value as the original portfolio and (2) a maturity date equal to the horizon date selected as the target date to which the duration of the immunized portfolio is fixed.

If the assets and liabilities of the financial institution have an imbalanced value, make fabricated zero-coupon bonds with a market value equal to the market value of the institution's equity and a maturity equal to the horizon date at which the equity's value needs to be guaranteed. To immunize the equity in this fashion over time, manage a self-financing investment that is long the assets, short the liabilities, and short the zero-coupon bonds, so that it maintains zero duration. DV01 techniques are just as appropriate as duration-based techniques for achieving immunization.

Contingent immunization sets a target value for the bond portfolio at the horizon date that is smaller than the face value of a zero-coupon bond. This target value is regarded as a floor below which the future value of the bond portfolio should not fall. The bondportfolio is managed actively without regard for duration until its value falls to a critical level.

Convexity measures how much DV01 changes as the yield of a bond or bond portfolio changes. If the convexity of a hedged portfolio is zero, even fairly large changes in interest rates over a short time span should not alter the value of the portfolio radically. If the convexity is large, constant monitoring of the bond prices might be a good idea. Most bonds have positive convexity. The convexity '*CONV(r)'* of a bond or a bond portfolio at a yield to maturity of *r* is the product of (1) €1,000,000 divided by the bond price and (2) the difference between the current DV01 (of the entire bond position) and the DV01 (of the entire bond position) at a yield of *r* + .0001; that is

$$CONV(r) = \left(\frac{€1 million}{P}\right)\left[DV01(r) - DV01(r+.0001)\right]$$

Convexity (unlike DV01, but like duration) is independent of the scale of the investment. The convexity of a portfolio of bonds is the value-weighted average of the convexity of each bond in the portfolio.

The assumption that interest rates are flat is a false portrait of realistic term structures and it is fraught with a number of pitfalls that are based on inherent flaws in logic.

Many practitioners have sought to develop the hedge with a variety of more sophisticated techniques. One of these employs the concept of a *yield beta*, which is the sensitivity of the hedge portfolio's yield to maturity to movements in the yield to maturity of the portfolio one is trying to hedge.

The better hedge is the hedge portfolio that makes the sum of

- the DV01 of the portfolio that one is trying to hedge
- the product of the DV01 of the hedge portfolio and its yield beta
- equal to zero.

If $DV01_H$ denotes the DV01 of the hedge portfolio, $DV01_P$ the DV01 of the original portfolio, and β the yield beta, the hedge solution is represented algebraically by

$$DV01_P + DV01_H \beta = 0$$

Though, this leaves open the question of how to compute β. More precise hedging requires a sophisticated term structure model. Price sensitivity is used in this kind of models.

The relationship between the price sensitivity and yield sensitivity to a factor is given by the equation:

$$\Delta P = \frac{\Delta r}{\Delta f} \times DV01$$

Where: ΔP = the change in the value of the fixed income security or portfolio

Δr = the change in its yield to maturity

Δf = the change in the interest rate factor that determines the term structure

This equation states that the change in the value of the security is the product of (1) the change in the yield to maturity with respect to a change in the interest rate factor and (2) DV01.

Hedging with *term structure* DV01 is the same as the yield beta method when the yield beta is constrained to be 1. the method also implies that the term $(\Delta r / \Delta f)$ in the equation above is the same for dissimilar maturity bonds.

Duration computed using te bond's yield to maturity for discounting, is known as the *MacAuley duration*. It is implicitly based on the assumption that yields for bonds of all maturities are identical.

Duration using weights on cash flow maturities obtained from discounting cash flows with zero-coupon bond yields is referred to as *present value duration*.

The two durations normally vary unless the bond is riskless and the yields to maturity of riskless cash flows of different maturities are the same.

With present value duration, there is no single interest rate to take a derivative with respect to. Each cash flow has its own interest rate for discounting. However, present value duration can be interpreted as the negative of the percentage change in the bond's value for a small parallel shift in the term structure of continuously compounded interest rates.

That is, defining r_t as the appropriate continuously compounded rate for cash flows *t* years from now and, assuming that for all *t*s, r_t shifts up by a constant δ, the derivative of the percentage change in the bond's price with respect to δ equals the negative of duration; that is: (present value) DUR = $-\frac{1}{P}\frac{dP}{d\delta}$

Term structure DV01 can be written as a constant times a derivative

(term structure) DV01 $= -.0001\frac{dP}{d\delta}$

Combining the last two equations implies

(term structure)DV01 = (present value) DUR $\times P \times .0001$