Chapter 2

2.1

We will discuss the multiple comparison procedure (MCPs), which involves comparisons among the group means. For situations where there are at least three groups and the ANOVA H0 has been rejected, some sort of MCP is needed to determine which means or combination of means are different. MCPs are only applicable for comparing levels of an independent variable that are fixed, so not for random-effects independent variables.

A contrast is a weighted combination of the means. Statistically this is defined as:

$$\psi_i = c_1\mu_{.1} + c_2\mu_{.2} + \ldots + c_J\mu_{.J} \quad (23)$$

cj represents contrast coefficients (or weights), which are positive, zero, and negative values used to define a particular contrast. Represents population group means.

The contrast is simply a combination of means of population groups a researcher wants to compare.

To form a fair or legitimate contrast, sum van cj =0 for the equal n's or balanced case. And sum (nj, cj) =0 for the unequal n's or unbalanced case.

When we test the contrast, the null and alternative hypotheses can be written as:

H_0: $\psi_i = 0$
H_1: $\psi_i \neq 0$

So we will test whether a particular combination of means are different. This also relates to the omnibus F test. The null and alternative hypotheses for omnibus F can be written in contrast as:

H_0: $all\ \psi_i = 0$
H_1: at least one $\psi_i \neq 0$

Contrasts can be divided into simple or pairwise contrasts (involving only two means), and complex or nonpairwise contrasts (involving more than two means). You can easily calculate how much possible unique pairwise contrast could be formed. It can be written as 0.5[J(J-1)]. So for J=3 (groups), there are 0.5[3(3-1)] = 3 possible unique pairwise contrasts.

The number of complex contrasts is greater than the number of simple contrasts when J is at least 4. The total number of unique pairwise and complex contrasts is [1+0.5(3J-1)-2J]. So for J=4 there are [1+0.5(34-1)-24] = 25 contrasts.

Many of the MCPs are based on the same test statistic, the "standard t":

$$t = \frac{\psi'}{s_{\psi'}} \qquad (24)$$

Where represents the standard error of the contrasts as follows:

$$s_{\psi'} = \sqrt{MS_{error} \sum_{j=1}^{J} \frac{c_j^2}{n_j}} \qquad (25)$$

The prime in this formula stands for that this is a sample estimate of the population value of the contrast. And nj stands for the number of observations in group j.

There are specific types of contrasts or comparisons. One way to divide the specific types is whether the contrasts are formulated prior to the research or following a significant F omnibus test.

Planned contrast (specific or a priori contrast). These are formulated prior the data collection. Mostly based on theory, previous research or specific hypotheses. The researcher is interested in certain contrasts, but not as in omnibus F test that examines all the contrasts. Fewer planned comparisons are usually conducted (due to their specify) than post hoc comparisons (generality). Planned contrasts are more specific and yield narrower confidence intervals, are more powerful and have a higher likelihood of a Type I error.

Post Hoc contrasts (unplanned or posteriori or post-mortem contrasts). This type is only done after statistically significant omnibus F test. In this case the researcher can take into account the family-wise error rate to achieve better overall Type I error protection.

As mentioned in post hoc contrasts, researcher may have to deal with family-wise Type I error rate. It depends on the MCP selected, but the researcher can deal with it by either setting alpha for each contrast or set for a family of contrasts. When alpha is set for each individual contrast, it is known as contrast-based (in other words per contrast). This alpha shows the probability of making a Type I error for that particular contrast.

When the alpha is set for a family it is known as family-wise.This represents the probability of making at least one type I error in the family or set of contrasts.

For orthogonal (independent or unrelated) the following property holds:

$$\alpha_{fw} = 1 - (1 - \alpha_{pc,})^{c} \qquad (26)$$

Where c= J-1 orthogonal contrasts. For nonorthogonal contrasts, this property is more complicated and we simply say:

$$\alpha_{fw} \leq c\alpha_{pc,} \qquad (27)$$

We can say a set of contrasts is orthogonal if thy are independent or unrelated (if the usual ANOVA assumptions are met) sources of variation. For J groups, you can construct J-1 orthogonal contrasts in a set. However there can be more than one set or orthogonal contrasts. With equal n's the two contrasts are defined to be orthogonal if the products of their contrasts coefficients sum to 0.

The following holds:

$$\Sigma_{j=1}^{J}(cjcj') = 0 \quad (28)$$

So orthogonally depends on the contrast coefficients, not on the group means.

For the case of unbalanced n's case, two contrasts are orthogonal if the following holds:

$$\Sigma_{j=1}^{J}\frac{c_j c_{j'}}{n_j} = 0 \quad (29)$$

Because of the denominator it is more difficult to find an orthogonal set of contrasts.

2.2

There are different types of MCPs. We will discuss a few; these are the best procedures in terms of ease of utility, popularity and control of Type I and Type II error. The first few procedures are for planned comparisons, the remainder is for post hoc comparisons.

Planned analysis of Trend

This is useful when the group has different quantitative levels of a factor, such as interval or ratio levels (i.e. age). We can see with this method whether the sample means vary with a change in the amount of the independent variable. The definition of trend analysis is: orthogonal polynomials and assume that the levels of the independent variable are equally spaced and that the number of observations per group is the same. Orthogonal polynomial contrasts use the standard t test statistic, which is compared to the critical values.

Orthogonal polynomial contrasts have two concepts, orthogonal contrasts (unrelated/independent) and polynomial regression. In polynomial regression there can be a linear trend (straight line), quadratic trend (curve with one bend, U-shaped) and cubic trend by a curve with two bends (S-shape). And from orthogonal contrasts we know for J groups, there can only be J-1 orthogonal contrasts in a set. From these two ideas we can conclude that the set of orthogonal contrasts can be formed where the first contrast evaluates a linear trend, the second a quadratic trend, and the third a cubic trend. So for J groups, the highest order polynomial that can be formed is J-1. The contrast coefficients for linear, quadratic and cubic trends are as follows:

More of those tables for more values of J can be found in table A.6.

	C_1	C_2	C_3	C_4
ψ_{linear}	-3	-1	+1	+3
$\psi_{quadratic}$	+1	-1	-1	+1
ψ_{cubic}	-1	+3	-3	+1

Mostly there is no researcher interested in polynomials beyond the cubic because they are difficult to understand and to interpret.

The standard error for the linear trend is computed with formula (25). The standard error for the quadratic trend is also computed with formula (25). Also for the standard error of the cubic trend we use this formula. It looks now that we do the same, but recall the contrast coefficients, so we use different number that we fill in into the formula. The t statistic is then calculated by using the contrast coefficients and the values of the means for the four groups (in this case):

$$t_{linear} = \frac{c1\overline{Y1}+c2\overline{Y2}+c3\overline{Y3}+c4\overline{Y4}}{s_{\psi'}} \qquad (30)$$

For example when the t statistic for linear trend is statistically significant (t test exceeds the critical value) but the higher-order trend is not significant, we will see a strong linear trend when we plot the profile.

There are some final remarks on orthogonal polynomial contrasts. First, stay between the range of levels investigated, because the trend may change outside this range. Second, in unequal n's case, it is difficult to formulate a set of orthogonal contrasts that make any sense to the researcher. Third, it needs to be taken into account in the contrast coefficients when the levels are not equally spaced.

Planned orthogonal contrasts (POC).

In this MCP the contrasts are defined ahead of time by the researcher and the set of contrasts are orthogonal. It is a contrast-based procedure. POC also uses the standard t test statistic that is compared to the critical value. Again the standard errors are computed with formula (25). And the test statistics are computed with formula (30).

However there is a practical problem with the POC procedure because the contrasts that are of interest to the researcher may not necessarily be orthogonal, or the researcher may not be interested in all of the contrasts of a particular orthogonal set. Another problem occur when the design is unbalanced, where an orthogonal set of contrasts may be constructed at the expense of meaningful contrasts. So:

- When you are interested in not orthogonal contrasts use another MCP
- If you are not interested in all of the contrasts of an orthogonal set, use another MCP
- If the design is not balanced and the orthogonal contrasts formed are not meaningful, then use another MCP.

Planned contrasts with Reference Group: Dunnett Method

It tests pairwise contrasts where a reference group is compared to each of the other J-1 groups. This method is a family-wise MCP and is slightly more powerful than the Dunn procedure (another planned family-wise MCP). The test statistic is the standard t. This is compared to the critical values from table A.7. However the standard error is simplified to:

$$s_{\psi'} = \sqrt{MS_{error}\left[\frac{1}{nc}+\frac{1}{nj}\right]} \qquad (31)$$

Where c is the reference group and j is the group to which it is being compared.

For the test statistic we use the same formula (30).

Planned Contrasts Dunn (Bonferroni) and Dunn-Sidak Methods

The Dunn method is a planned family-wise MCP. It tests either pairwise or complex contrasts for balanced and unbalanced designs. It uses the standard t test statistic with one important exception. The alpha level is split up among the set of planned contrasts. Typically the per contrast alpha level is set at alpha/c , with c as number of contrasts.

$$(\alpha_{pc} = \alpha_{fw}/c).$$

This method uses the standard t test statistic that is compared to the critical t values for a two-tailed test from table A.8.The standard errors are computes with formula (25). And again the test statistic is computed with formula (30). So with the same dataset, the t-values will not change from the POC method. However, the critical t-values will change. In the Dunn method the critical values are larger than the POC method, which makes is more difficult to reject H0.

This method is slightly conservative (not as powerful), a modification is known as the Dunn-Sidak procedure is less conservative, and uses slightly different critical values.

Complex Post Hoc Contrasts: Scheffe and Kaiser-Bowden Methods

The Scheffe procedure can be used for any possible comparison, orthogonal or nonorthogonal, pairwise or complex, planned or post hoc, where the family-wise error rate is controlled. However the test is very general, which makes it less powerful. The Scheffe method is only recommended for complex post hoc comparisons. This method is the only MCP that is necessarily consistent with the results of the F ratio in ANOVA. For example when the F ratio is not statistically significant, none of the contrasts in the family will be significant with the Scheffe method. Again we use the standard t test statistic and it is compared with the critical value obtained from the F table in Table A.4. These critical values are large and that makes the procedure less powerful. Again the standard error is computed by formula (25). And the test statistics with formula (30).

The results are the same as the results of the Dunn procedure, however the critical values differ. From this we can again conclude that the critical values are larger and thus this test is more conservative (less powerful). A modification of the Scheffe method is the Kaiser-Bowden method. This is used for groups with unequal variances.

Simple Post Hoc Contrasts: Tukey HSD, Tukey-Kramer, Fisher LSD, and Hayter Tests.

The Tukey's honestly significant difference (HSD) test (studentized range test) is one of the most popular post hoc MCPs. It is a family-wise procedure and is most useful for looking at all the pairwise contrasts with equal n's per group. For this approach, the first step is to rank the means from largest to smallest. The test statistic is computed as follows:

$$q_i = \frac{\bar{Y}_{.j} - \bar{Y}_{.j'}}{s_{\psi'}} \qquad (32)$$

where

$$s_{\psi'} = \sqrt{\frac{MS_{error}}{n}} \qquad (33)$$

i identifies the specific contrast

j and j' show the two groups means to be compared

n represents the number of observations per group (equal n's per group is required).

This test statistic is compared to the critical value in table A.9. Where the degrees of freedom are equal to J(n-1). We start by comparing the largest pairwise difference in the set of J means. If this is not significantly different, we stop the analysis because no other pairwise difference can be significant. When it is significant, we will continue the analysis until a nonsignificant difference is found.

The HSD test has exact control of the family-wise error rate assuming normality, homogeneity and equal n's. This method is more powerful than the Dunn or Scheffe method for testing for all possible pairwise contrasts. However the Dunn method is more powerful when testing less than all possible pairwise contrasts. This method is recommended when testing a pairwise contrast in equal n situation.

So we need another test for the unequal n' situation. There are several alternatives. These include the Tukey-Kramer modification. This test also assumes normality and homogeneity. The test statistic is the same as the HSD, but the standard error is computed differently:

$$s_{\psi'} = \sqrt{MS_{error}\left[\frac{1}{2}\left(\frac{1}{n_1} + \frac{1}{n_2}\right)\right]} \qquad (34)$$

The critical value is determined the same way as the Tukey HSD procedure.

Another alternative is the Fisher's least significant difference (LSD) test (protected t test). It is a sequential procedure where the significant ANOVA F is followed by the LSD test in which all pairwise t tests are examined. The t test statistic is again compared to the critical value. This LSD test had precise control of the family-wise error rate for the three-group situation. However for more than three groups, the protection decreases rapidly. For the case there are more than three groups, the modification the Hayter test, is useful. This had more power than the Tukey HSD and has excellent control of family-wise error.

Simple Post Hoc Contrasts for Unequal Variances: Games-Howel, Dunnet T3 and C Tests.

These tests are useful for unequal group variances. The Dunnet T3 is recommended when n<50, the Games-Howell is recommended for n>50. The C tests perfom about the same as the Games-Howell tests.

These follow ups are all based on the ANOVA test. However we also have seen the Kruskal-Wallis test, which is the nonparametric equivalent to ANOVA. There are some post hoc procedures that are useful to follow up statistically significant overall Kruskal-Wallis test. These are the nonparametric equivalents to the Scheffe and Tukey HSD methods. The pairwise or complex contrasts can be formed as in the parametric case. The test statistic is Z and is computed as follows:

$$Z = \frac{\psi'_i}{s_{\psi'}} \qquad (35)$$

Where the standard error in the denominator is computed as:

$$s_{\psi'} = \sqrt{\frac{N(N+1)}{12} \sum_{j=1}^{J} \left(\frac{c_j^2}{n_j}\right)} \qquad (36)$$

In this formula N is the total number of observations. For the Scheffe method, the test statistic Z is compared to the critical value obtained from the chi 2 table form Table A.3. For the Tukey HSD procedure, the test statistic is compared to the critical value obtained from the values for the studentized range statistics in table A.9.

2.3

We will now show the steps for a Tukey HSD in SPSS (see page 73-75 for outcome):

- From "Univariate" dialog box, click on "Post Hoc" to select various post hoc MCPs. Or you can click on "Contrast" to select various planned MCPS.
- (Post Hoc MCP). Move the independent variable form the "Factor(s)" list box to the "Post Hoc Tests for" box. Check the appropriate MCP for your situation. (We will select "Tukey"). Click on "continue" to return to original dialog box and click on "OK" to return to generate output.
- (Planned MCP). To obtain trend analysis contrasts, click "Contrasts" bottom from the "Univariate" dialog box. Click "Contrasts" pull-down and scroll down to "Polynomial".
- Click "Change" to select "Polynomial" and move it to be displayed in the parentheses next to the independent variable. This type will allow testing of linear, quadratic and cubic contrasts. Then click "continue" and return to "univariate" dialogue box.