Chapter 4: Portfolio

A *portfolio* is a package of different investments. Theory about this phenomenon suggests that investors are *mean-variance optimizers*: seekers of portfolios with the lowest possible return variance for any given level of mean return. *Mean-variance analysis* describes mathematically how the risk of individual securities contributes to the risk and return of portfolios. *Diversification* is holding many securities to lower the risk.

The *portfolio weight* for stock j, *Xj*, is the proportion of stock j to the protfolio; that is:

Xj = Dollars held in stock j / Dollar value of the portfolio

The sum of the ortfolio weights must be equal to 1.

When an investor sells short, he sells an investment that he does not own. We can also say that he takes a short position in the security. When you sell short, you place a negative portfolio weight on a security. When you buy an investment, you take a long position. A long position always has a positive portfolio weight. *Feasible portfolios* are the set of portfolios that one can invest in.

There are two identical methods to compute portfolio returns.

*Ratio method: (*Value portfolio end period / Value portfolio beginning period) -1

Portfolio-weighted average method: This method fist assigns weights to the stocks and then sums up the weighted returns.

For *N* stocks the portfolio return formula becomes:

$$\sum_{i=1}^{N} \chi_i r_i$$

To calculate the expected return, weight each of the return outcomes by the probability of the outcome and sum the probability-weighted returns over all outcomes.

The expected returns for a portfolio of two stocks is:

$$E(R_p) = E(x_1 r_1 + x_2 r_2) = x_1 E(r_1) + x_2 E(r_2)$$

Leveraging an investment is selling short an investment with a low expected return and using the payoffs to increase a position in an investment with a higher expected return.

The expected portfolio return is the portfolio-weighted average of the expected returns of the individual stocks in the portfolio:

$$R_p = \sum_{i=1}^{N} x_i r_i$$

Risk aversion is the anxiety investors have for losses. The *variance* of a return is the expected value of the squared demeaned return outcomes:

$$\text{var}(r) = E[(r - \bar{r})^2]$$

r = return on investment

$\bar{r}$ = expected return

The *standard deviation* is the square root of the variance, denoted σ .

A positive correlation or covariance between two outcomes is a sign that the two outcomes move together. When computing the *covariance, you measure* the relatedness between units.

The covariance between two returns is:

$$\sigma_{12} = E[(r_1 - \bar{r}_1)(r_2 - \bar{r}_2)]$$

Computing the covariance is only possible when there is information about how the various outcomes pair up, the *joint distribution*. To compute a covariance, find out the probability-weighted average of the product of the two demeaned returns linked with each of the paired results using the joint distribution, see example 4.10 for calculation.

The *correlation* between two returns, ρ ,is the covariance divided by the product of their standard deviations:

$$\rho(r_1, r_2) = \frac{\text{cov}(r_1, r_2)}{\sigma_1 \sigma_2}$$

The formula for translating correlations into covariances is:

$$\text{cov}(r_1, r_2) = \rho(r_1, r_2)\sigma_1 \sigma_2$$

Variance of a portfolio of two investments is:

$$= x_1^2 \sigma_1^2 + x_2^2 \sigma_2^2 + 2x_1 x_2 \sigma_{12}$$

When two stocks have positive portfolio weights, then the lower the correlation is, the lower the variance is.

The formula for the variance of a portfolio return is:

$$\sigma_p^2 = \sum_{i=1}^{N} \sum_{j=1}^{N} x_i x_j \sigma_{ij}$$

We can also compute the covariances when only the correlations and the variances or standard deviations are given:

$$\sigma_p^2 = \sum_{i=1}^{N}\sum_{j=1}^{N} x_i x_j \rho_{ij} \sigma_i \sigma_j$$

The formula to compute the covariance of the return of a portfolio with the return of stock k:

$$\sigma_{pk} = \sum_{i=1}^{N} x_i \sigma_{ik}$$

The *mean-standard deviation diagram* shows how investors should view the trade-offs between means and variances when selecting portfolio weights for their investment decisions. It plots the mean return on the Y-axis and the standard deviations of returns on the X-axis.

The equation for a portfolio return combining a risk-free asset and a risky investment yields:

$$\overline{R_p} = r_f + \frac{\overline{r_2} - r_f}{\sigma^2} \sigma_p$$

This equation is a straight line with a slope of $(\overline{r_2} - r_f) / \sigma_2$.

When investors buy money at the risk free rate to invest, with that money, in a risky asset, then the expected return will be lower and the standard deviation of the portfolio will be higher.

It is possible to eliminate risk with a portfolio of two perfectly correlated stocks. Therefore it is essential to be long in one investment and short in the other. When two investments are perfectly negatively correlated, the investor can get rid of the variance by being long in both investments.

We can interpret the covariance as the marginal variance of a stock. The marginal variance is the amount of change in the variance of a portfolio, when the portfolio weight on stock increases by a bit.

The portfolio of a group of stocks that minimizes return variance is the portfolio with a return that has an equal covariance with every stock return.