Chapter 5: CAPM

The *feasible set* of the mean-standard diagram is the set of mean and standard deviation results, that are carried out from all feasible portfolios.

The mean-variance analysis makes use of two assumptions:

- When investors make decisions today, they only pay attention to the means and the variances of the returns of their portfolio over a certain period. We also think of investors as risk averse.
- Financial markets are frictionless. This means that all investments are tradable at any price and in any quantity (positive and negative).

The *efficient frontier* shows what the most efficient trade-off is between the mean return and the variance.

With two-fund separation we divide the returns of all mean-variance efficient portfolios into weighted averages of the returns of two portfolios.

The *tangency portfolio* is the unique optimal portfolio that does not invest in the risk-free asset.

The *capital market line (CML)* stands for the portfolios that combine all investments at best. All investors will invest in portfolios on the capital market line when there is a risk free asset and when we hold on to the assumptions of the mean-variance analysis.

The capital market line formula:

$$\bar{R}_p = r_f + \frac{\bar{R}_T - r_f}{\sigma_T}\sigma_p$$

where $\bar{R}_T$ and σ_T are, respectively, the mean and standard deviation of the tangency portfolio's return and r_f is the return of the risk-free asset. Ri – Rf is de risk premium.

$$\frac{\bar{r}_i - r_f}{\text{cov}(r_i, R_t)}$$

$\bar{R}_T$ is the same for all stocks.

To compute the weights of the tangence portfolio:

Find the portfolio weights (the sum don't have to be 1) that make the covariance of the portfolio with each stock equal to their risk premiums

After this you have to rescale the weights. This time it has to sum up to 1.

One difficulty in applying mean-variance analysis is that means and covariances are most of the time unobservable. This barrier in applying mean-variance analysis to find out the efficient portfolios of individual stocks can be overcome with additional assumptions.

The equation below describes the relation between the expected return of an investment and the covariance between the returns of the tangency portfolio and the investment:

$$\bar{r} - r_f = \frac{\text{cov}(r, R_T)}{\text{var}(R_T)}(\bar{R}_T - r_f)$$

The covariance between the returns of the tangency portfolio and the investment divided by the variance of the tangency portfolio is de Beta, denoted by β . We can now reshape the equation in:

$$\bar{r} - r_f = \beta(\bar{R}_T - r_f)$$

The *securities market line (SML)* stands for the link between the beta and the mean return of an investment.

We can also compute the beta by summing up the portfolio weighted averages of the betas of single securities.

Beta is just the covariance divided by the same constant for every stock.

Tracking portfolios are portfolios that copied the return characteristics of huge portfolios using a relatively small number of stocks. When the tracking portfolio has the same marginal variance as the stock, then the stock and its tracking portfolio also must have the same expected return.

The *Capital Asset Pricing Model (CAPM)* is a model that shows the link between the expected returns and the risks. We already made some assumptions, but when applying the CAPM, a third assumption is needed.

3. All investors have the same ideas and draw the same conclusions about the means and standard deviations of all feasible portfolios.

The *market portfolio* is a portfolio where the weight on each asset is the market value of that asset divided by the market value of all risky assets.

A *value-weighted portfolio* means that the portfolio weight on each of its stocks is proportional to the market value of that stock.

When we hold on to the assumptions of the CAPM, and if a risk-free asset exists, the market portfolio is the tangency portfolio and the expected returns of financial assets are determined by:

$\bar{r} - r_f = \beta(\bar{R}_M - r_f)$ where $\bar{R}_M$ is the mean return of the market portfolio, and β is the beta computed against the return of the market portfolio.

Furthermore, if a risk-free asset is present, every investor should optimally hold a combination of the market portfolio and a risk-free asset.

Conducting Tests to the CAPM may have some difficulties because the market portfolio is not directly observable. Applications of the theories use a range of proxies for the market. Although the results of empirical tests of the CAPM that use these proxies cannot be evaluated as conclusive, they provide valuable insights about the appropriateness of the theory as implemented with the specific proxies used in the test.

When testing the CAPM a two stage approach was used. First, the betas were estimated with a set of time-series regressions, one for each security.

Each of these regressions, one for each security *j*, can be represented by the following formula:

$$r_{jt} = \alpha_j + \beta_j R_{Mt} + \in_{jt}$$

where: α_j = the regression's intercept

β_j = the regression's slope coefficient

r_{jt} = the month *t* return of stock *j*

R_{Mt} = the month *t* return of the value-weighted portfolio of NYSE and AMEX stocks

$\in_{jt}$ = the month *t* regression residual for stock *j*

The second step searched out estimates of the intercept and slope coefficient of a single *cross-sectional regression*, in which each data observation fits to a stock. Represented by equation:

$$\bar{r}_j = \gamma_0 + \gamma_1 \beta_j + \gamma_2 CHAR_j + \delta_j$$

where: $\bar{r}_j$ = average monthly historical return of stock *j*, each *j* representing a NYSE-listed stock

β_j = estimated slope coefficient from the time series regression

$CHAR_j$ = a characteristic of stock *j* unrelated to the CAPM

γ_s = intercept and slope coefficients of the regression

δ_j = stock *j* regression residual

If the CAPM is true, the second step regression, should have the following aspects:

the intercept, γ_0, should be the risk-free return

the slope, γ_1, should be the market portfolio's risk premium

γ_2 should be zero since variables other than beta, represented as $CHAR_j$, should not explain the mean returns once beta is accounted for.

A second set of CAPM tests, by Black, Jensen and Scholes (1972), examine the restrictions on the intercepts of time-series market model regressions. Consider the regression:

$$r_{jt} - f_f = \alpha_j + \beta_j (R_{Mt} - r_f) + z_{jt}$$

The CAPM indicates that the intercept, α_j, is zero for every stock or portfolio.

The most important violations of the CAPM are:

- the relation between estimated beta and average historical return is much weaker than the CAPM puts forward.
- The market capitalization or size of a firm is a predictor of its average historical return. This relation cannot be accounted for by the fact that smaller capitalization stocks tend to have higher betas.
- Stocks with low market-to-book ratios have higher returns than stocks with high market-to-book ratios most of the time.

Again, discrepancies in beta do not explain this difference.

Stocks that have performed well over the past six months have higher expected returns over the following six months, in most of the cases.

There are two explanations for the poor achievements of the CAPM to explain average stock returns. The first has to do with the chance that the different proxies for the market portfolio do not capture all of the relevant risk factors in the economy.

For example, *human capital*, which is the present value of a person's future wages, is not acquired. The second clarification is that it is only a false theory because investors have behavioral biases against classes of stocks that have nothing to do with the mean and marginal risk of the returns on stocks.