Chapter 6: Factor Models

Factor models are formulas that divide the returns of securities into two elements:

1. The *common factors*, which events in the economy that affect a large number of different investments.
2. A risk component that is personal to the investment

Firm-specific components only have influence on the firm itself. *Firm-specific risk* is the risk of a security that is created by the firm-specific components. Stocks are influenced by many sources of *factor risk*. Factor risk is the variety in returns that is created by common factors.

Arbitrage pricing theory (APT), relates the factor risk of an investment to its expected rate of return. It is termed the arbitrage pricing theory because it is based on the principle of no arbitrage. We speak of an arbitrage opportunity when somebody can make an investment and earn money without having risks.

A *one-factor model* is also called the market model because there is only one factor, which is the market factor.

The *systematic (market) risk* of a security is the portion of the security's return variance that is explained by market movements. The *unsystematic risk* is the risk that cannot be explained by market movements.

We can also diversify risk by holding many securities in a portfolio. The risk that we can eliminate by holding such a portfolio is *diversifiable risk*. *Nondiversifiable risk* cannot be eliminated.

The *R-squared* determines the proportion of the return variance that is due to systematic risk. The formula:

$r_i = \alpha_i + \beta_i R_M + \in_i$ in which the *i* is an arbitrary stock and R_M is an arbitrary market index with return R_M.

A *multifactor model*, is a model with more than one common factor:

$$r_i = \alpha_i + \beta_{i1} F_1 + \beta_{i2} F_2 + ... + \beta_{iK} F_K + \in_i$$

Each common factor is denoted by *F*, for which different stocks have different beta's. They also have the firm-specific components, which are uncorrelated.

There are three ways to estimate the common factors in a factor model:

- Use a statistical method to determine *factor portfolios*. *Factor analysis* is a statistical method based on the idea that the covariances between stock returns provide information that can be used to decide on the common factors that produce the returns. The factors and factor betas produced by factor analysis are those that give the best feasible explanation of the covariances estimated from historical stock returns.

  A pro of forming factors is that the factors selected are those that do the best job in explaining the covariances between all stocks. A disadvantage is that is gives not much insight into which economic variables the factors are linked. The second problem is that the technique assumes that the return covariances are staying constant over time.

- Use macroeconomic variables for the factors. This approach makes use of macroeconomic variables, like inflation and the interest rate, and determines which variables best explain the observed pattern of stock returns.

One great advantages of the macroeconomic variables is that it names the factors. With this managers can get a good economic intuition about the sources of risk. However, it is hard to measure the unexpected changes in the macroeconomic variables. Also a downside is that some important factors are hard to quantify.

- Make use of firm characteristics

When selecting the characteristics, one should look after what common characteristics makes stocks go up and down. An advantage of this approach is that it does not require the covariances to be constant. A downside of this approach is that factors are chosen because they explain historical ''accidents'' but they will not explain future expected returns.

The size of a security's factor beta describes how sensitive the security's return is to changes in the common factors.

We explained earlier that the factor beta of a portfolio is the weighted average of individual securities betas on that factor. Also the alpha and the epsilons of the portfolio can be computed in this way.

Covariances can be calculated with the following formulas:

$$r_i = \alpha_i + \beta_{i1} + \beta_{i2}F_2 + ... + \beta_{iK}F_K + \in_i$$
$$r_j = \alpha_j + \beta_{j1} + \beta_{j2}F_2 + ... + \beta_{jK}F_K + \in_j$$

Then the covariance between r_i and r_j is:

$$\sigma_{ij} = \beta_{i1}\beta_{j1}\ \text{var}(F_1) + \beta_{i2}\beta_{j2}\ \text{var}(F_2) + ... + \beta_{iK}\beta_{jK}\ \text{var}(F_K)$$

When the returns of stocks have approximately similar betas then the stocks are likely to be highly correlated with each other. And when the betas are very different then there is less correlation.

To create a tracking portfolio, use the following serie of steps:

- Find out the number of relevant factors
- Determine the factors and calculate the factor betas
- Set up one equation for each factor beta.
- Work out the equations for the tracking portfolio's weights and the weights must sum to 1.

Pure factor portfolios are portfolios that have no firm-specific risk. They have a sensitivity of 1 to one of the factors and 0 to the others.

In a *K*-factor model, it is possible to construct *K* pure factor portfolios, from any *K + 1* investments that lack firm-specific risk. Example 6.6 explains how to do this.

The several risk premiums of the *K* factor portfolios in a *K*-factor model are normally represented as $\lambda_1, \lambda_2, ..., \lambda_K$. The expected return of a factor portfolio is $r_f + \lambda_K$.

A sufficiently large number of securities in the portfolio makes it likely that the portfolios will have negligible firm-specific risk, so it is generally possible to construct portfolios that perfectly track investments that have no firm-specific risk, by forming portfolios of the pure factor portfolios that have the same factor betas as the investment one wishes to track.

The expected return on this tracking portfolio is:

$r_f + \beta_{i1}\lambda_1 + \beta_{i2}\lambda_2 + ... + \beta_{iK}\lambda_K$, where $\lambda_1, \lambda_2, ..., \lambda_K$ denote the risk premiums of the factor portfolios and r_f is the risk-free return.

The APT needs four assumptions:

- Returns can be described by a factor model.
- The financial markets are frictionless.
- There are a large number of securities, so that it is possible to form portfolios that diversify the firm-specific risk of individual stocks => firm-specific risk does not exist.
- There are no arbitrage opportunities.

An arbitrage opportunity exists when the following equation does not solve:

$$\bar{r}_i = r_f + \beta_{i1}\lambda_1 + \beta_{i2}\lambda_2 + ... + \beta_{iK}\lambda_K$$

The graph of the formula in a one-factor case is almost the same as the graph of the securities market line. If there is no arbitrage, all investments must lie on this line. In a two-factor case, the formula graphs as a plane in three dimensions. The location and slope of the plane are determined by the risk-free return, which is the height of the plane above the origin, and the two risk premiums. All investments must lie on this plane if there is no arbitrage.

When you want to know if the APT holds, find out whether a single set of λ 's can generate the expected returns of all the securities.

Tests of the APT analyse the following three implications:

The expected return of any portfolio with factor betas that are all equal to zero is the risk-free rate.The expected returns of securities increase linearly with increases in a given factor beta. No other characteristics of stocks, other than factor betas, determine expected returns.