

Chapter 7: Measuring productivity

Recall from Chapter B, when we introduced the production function, which describes the produced output as a function of total productivity factors used.

Productivity is therefore the effectiveness with which the factors of production were converted into the final product/output.

Productivity in the production function

Since the production function describes the relationship of input factors and produced output, the function can be sketched in a graph, with the y-axis representing output (in this case per worker) and on the x-axis the factors (per worker). This graph allows us to assess 3 different reasons for output differences between two countries (let's call them A and B).

The slope of the production function represents the efficiency (i.e. the productivity), because a higher slope, i.e. higher productivity will produce more output by the same input used.

There are three possible sources of differences in output per worker. (See figure 7.1 on page 201)

1. Country 1 and Country 2 are sharing the same production function (e.g. also same productivity). If Country 1 is more abundant in factors of production per worker it is therefore able to produce more output than Country 2. The higher factor level leads to a higher intersection on the productivity function and therefore also a higher output level. Therefore, the gap between the output levels between the two countries is caused by factor accumulation in country 1.
2. Both economies are facing different production functions, with different slopes (i.e. productivities) but the factors of production per worker are identical. Country 1 has a production function with a steeper slope than Country 2 and is therefore able to convert the same level of input factors into a higher output. According to the production equation (when holding the factors of production constant) the increase of output must be due to a higher productivity.
3. In the third scenario, both starting scenarios from scenario 1 and 2 are combined; Country 1 has a more productive production function (i.e. steeper slope) and a higher production factor accumulation. Therefore, also a combination of differences in production function and input factor availability can lead to a difference in output.

Thus, the possible sources of differences in output between countries can be based on different factor availability, different productivity (higher production function slope) or both combined.

Comparing the level of Productivity among countries

In order to compare quantitative output differences between countries, caused by productivity, we will use the Cobb-Douglas function.

Please note that we inserted h as a measurement for the quantity of human capital. If we convert the Cobb-Douglas function in per worker terms, we divide it by "L" and therefore get:

$$y = A(K^\alpha h^{1-\alpha}) \quad (\text{output per worker} = \text{Productivity (A)} * \text{Factors of production})$$

To compare the level of output of countries we divide the formula of the first country by the other. That results in the following equation:

$$\text{Output ratio} = \text{productivity ratio} * \text{ratio of factors of production}$$

This shows that countries can differ in their levels of output because of differences in productivity, factor accumulation or both. The larger the ratio of output in two countries, the larger a productivity gap we would infer. The larger the gap in accumulation of factors, the smaller the productivity gap we would infer.

After rearranging that term we find an equation to compare the level of productivity. We cannot measure productivity directly but with this formula we can measure it indirectly;

$$\text{Productivity ratio} = \text{Ratio of output} / \text{Ratio of factors of production}$$

This formula can be formally written as:

$$\frac{A_1}{A_2} = \frac{\frac{y_1}{k_1^\alpha h_1^{1-\alpha}}}{\frac{y_2}{k_2^\alpha h_2^{1-\alpha}}}$$

Using this formula is known as **development accounting**: The technique for breaking down differences in income into the part that is accounted for by differences in productivity and the part accounted for by differences in factor accumulation.

Differences in the growth rate of productivity among countries

From the previous equations we can derive an equation for the growth rate of productivity:

$$\text{Growth rate of productivity} = \text{growth rate of output} - \text{growth rate of factors of production}$$

We can transform this formula into symbols, where the subscript g stands for the growth rate of that variable;

$$A_g = y_g - \alpha k_g - (1-\alpha) h_g$$

Using the technique from this formula is called **growth accounting**.