Chapter 7: Derivative pricing

A *derivative* is a financial instrument whose value today or at some future date is derived fully from the value of another asset, which is the *underlying asset*.

When a *forward contract* is closed, there is the obligation to buy or sell a security or commodity at a preset price at some future date. The exchange takes place at the *maturity date*. The exchange will take place at maturity. *Futures contracts* are a special type of contracts. It differs from the forward in the timing of the cash flows. As we said before, the cash in a forward is paid at maturity while with futures the cash paid at maturity is the final futures price. The daily profit or loss is paid on a daily basis. We call this procedure in futures marking to market. An advantage of this is that it decreases the risk of default. *Open interest* is the number of contracts outstanding. An example of this is the number of bets investors made on the oil price.

A *swap* is an understanding between two investors, to periodically exchange the cash flows of one security for the cash flows of another. The swap maturity is the last dat of exchange. The *notional amount* is the amount of money on which interest is computed.

The cash flows in an interest rate swap are netted so that from time to time, only one of two investors pays cash. A currency swap exchanges cash flow streams in two different currencies.

Zero-cost instruments are contracts where the parties can enter the contract without having to pay money to each other.

When you have an *option*, you have the right to buy an underlying security at a preset price. The value of an option is the difference between the stock price and the strike price. This value determines whether you exercise an option or not. When an option is in the money, its current price is higher than its strike price. *Out of the money* is the opposite. *At the money* means that the current price and the strike price are the identical.

Warrants are a kind of options which an organization can issue on their own stock. Warrants and other types of call options differ the most in that the company issues additional stock when an investor exercises a warrant at strike price. Exercise of warrants involves the issuance of new shares of stock at bargain prices.

The valuation of derivatives has two elements:

1. Perfect tracking portfolios
   You can always make a perfect tracking portfolio. It has to contain the risk free asset and the underlying asset.

2. No arbitrage and valuation
   We can speak of arbitrage when it costs more to buy the derivative than the tracking portfolio or vice versa.

The no-arbitrage value of a forward contract on a share of stock is (we assume that the stock pays no dividend prior to T):

$$S_0 = \frac{K}{(1+r_f)^T}$$ where, S_0 = current price of the stock

The forward price is:

$$F_0 = S_0(1+r_f)^T$$

We make use of Binomial pricing models when the price of the underlying asset is obvious at a future date. The dynamic strategy of tracking and valuation requires that the holdings in the risk free asset and the underlying asset change very often. This in order to track the futures payoffs of a derivative perfectly.

When the price of an underlying security only can take two values at the next trading point, we call it a binomial process. We call this two values the up state and the down state.

We can determine the perfect tracking portfolio , with binomial processes, by solving two equations in two unknowns. The up node formula is: $\Delta S_u + B(1+r_f) = V_u$. This equation has to paired with the equation for the down node: $\Delta S_u + B(1+r_f) = V_d$

Where Δ = number of units (shares) of the underlying asset

B = number of dollars in the risk-free security

r_f = risk-free rate

S_u = value of the underlying asset at the up node

V_u = value of the derivative at the up node

V_d = value of the derivative at the down node

To find the value of the derivative, make use of the following formula:

$$V = \Delta S + B$$

The probabilities and the risk aversion are not taking into account is this method, because that information is already captured by the price of the underlying asset.

A method for valuing derivatives is the risk-neutral method. This method contains three steps:

- Identify the risk-neutral probabilities. Risk-neutral probabilities are a set of weights applied to the future values of the underlying asset along each path.
- Multiply each risk-neutral probability by the related future value for the derivative and sum the products.
- Divide the sum of the products by the risk free rate.

This is a shortcut for going through the tracking portfolio method's valuation steps.

The risk-neutral probabilities, π , must solve:

$\pi u + (1-\pi)d = 1 + r_f$ where u = 1 + per period rate of the underlying asset at the up node

d = 1 + per period rate of the underlying asset at the down node

The no-arbitrage futures price is the same as a weighted average of the expected futures prices at the end of the period, where the weights are the risk-neutral probabilities;

$F = \pi F_u + (1-\pi)F_d$ where F is the current futures price

The equation to determine the risk-neutral probabilities, π and 1 - π from futures prices:

$$\pi = \frac{F - F_d}{F_u - F_d}$$

The formulas of the two-period binomial valuation are:

$$V_u = \frac{\pi V_{uu} + (1-\pi)V_{ud}}{1+r_f} \qquad V_d = \frac{\pi V_{ud} + (1-\pi)V_{dd}}{1+r_f}$$

The natural logarithm of the return of a security is normally distributed when the price movements of the security are determined by the lognormal distribution.

Once the annualized standard deviation, σ , of this normal distribution is known,

u and d are estimated as follows: $u = e^{\sigma\sqrt{T/N}}$ $d = \frac{1}{u}$

Where T = number of years to expiration

N = number of binomial periods

e = exponential constant

Thus $\sqrt{\frac{T}{N}}$ = square root of the number of years per binomial period

We also know a method called simulation. The computer gives some random numbers which are used to generate outcomes. These outcomes are averaged to obtain values.