Lecture 7

Chapter 15

Today, we focus on the role of technological progress, and focus again on the Solow growth model. We will see that sustainable economic growth is due to technological growth, the endogenous variable in the AK model.

The Solow model
The Solow model explains economic growth in the long run with a production function that has constant returns to scale and decreasing returns to production factors. Last week we assumed that there was no technological progress and no population growth. These assumptions led us to the conclusion that capital accumulation (through saving) can cause economic growth, but it cannot lead to sustained economic growth. Today, we will see that technological progress leads to sustained economic growth in the steady state.

We can write the production function $Y = F(K, N, A)$ as $Y = F(K, AN)$.

Y = output
K = capital
N = labour
A = state of technology
F = how much is produced given the amounts of K, N and A

To focus on the effect of capital accumulation, we divide both sides of the equation by N
$\frac{Y}{N} = F\left(\frac{K}{N}, 1\right)$
As long as there is capital accumulation, there will be growth. Until we arrive in the steady state, where there will be no growth anymore.

The coup douglas production function is given by $Y = K^{\alpha}(AK)^{1-\alpha}$, with decreasing returns to effective labour. We will now focus on growth per effective worker. Capital per effective worker is how much capital is there for a worker who can use technology. With more technology, every worker can produce more.
$\frac{Yt}{N} = f\left(\frac{Kt}{N}\right)$

An example:
$\frac{Y}{N} = \frac{K^{0,5}N^{0,5}}{N} = \frac{K^{0,5}}{N^{0,5}} = \left(\frac{K}{N}\right)^{0,5}$

Memory week 6
To see how output and investments are related, we make the assumptions that the economy is closed (NX = 0), there is no government, which means that public saving T-G = 0 and that private saving is proportional to income, so S = sY. This gives us $I_t = sY_t$. Let us verify:

Y = C + I + G + NX
G = 0 and NX = 0, so what is left is Y = C + I where C stands for households who consume and save and I stands for investing firms.

C = cY
S = (1 – c)Y
s = 1 – c
Combining the latter two gives S = sY

Y – C = I
S = I (equilibrium in the goods market)

sY = I
Now we verified this we see that investment is a fixed share of output.

The evolution of the capital stock is given by $K_{t+1} = (1-\delta)K_t + I_t$, where δ denotes the rate of depreciation. Using our assumptions made last week we can write

$$\frac{K(t+1)}{N} = (1-\delta)\frac{Kt}{N} + s\frac{Yt}{N}$$

Or:

$$\frac{K(t+1)}{N} - \frac{Kt}{N} = s\frac{Yt}{N} - \delta\frac{Kt}{N}$$

The change in the capital stock per worker (left side) is equal to saving per worker minus depreciation (right side).

s = the savings rate. The higher the savings rate, the more investment you have.
When I increases sY also increases.
The higher the rate of depreciation, the lower is the capital.

We can express output per worker $\left(\frac{Yt}{N}\right)$ in terms of capital per worker

$$\frac{K(t+1)}{N} - \frac{Kt}{N} = sf\left(\frac{Kt}{N}\right) - \delta\frac{Kt}{N}$$

Change in capital from year t to year t+1 = investment in year t – depreciation in year t

The steady state

This week we define the steady state as the level of capital stock per worker for which the capital stock per worker is not going to change anymore. We will arrive in the steady state if the equation above equals 0, so when $\frac{K(t+1)}{N} - \frac{Kt}{N}$ = 0. If this is 0, $sf\left(\frac{Kt}{N}\right)$ must equal $\delta\frac{Kt}{N}$. This is the steady state condition.
It is not hard to calculate the steady state because the savings rate, the rate of depreciation and the production function are given. So you only have to calculate (K/N)

In the graph on slide 11 we see that the green line is lower than the blue line. This is due to the savings rate which is between 0 and 1.

If the investment per worker is higher than the depreciation per worker there will be an increase in the capital stock until we are in the steady state. You can see it in the graph by the arrows pointing to the right. In the steady state the capital stock is not changing anymore. As long as you are on the left side of the steady state there will be an increase of capital, so an increase of K. Because of an increase in K, there will be an increase in Y and this is defined as economic growth. There will only be economic growth if we are not in the steady state because if capital is not changing anymore, Y is not changing anymore. So output is constant in the steady state.
If we are on the right side of the steady state the depreciation per worker is higher than the investment per worker. Now we are dealing with negative growth until we are in the steady state. You can see it in the graph by arrows pointing to the left.

Labour and effective labour

Labour is represented by N, effective labour is represented by AN. In the production function $Y = F(K, AN)$ an increase of A can mean two things:

- Output decreases
- Less labour is required to get the same output because labour is becoming more efficient, i.e. effective labour

The relation between output per effective worker and capital per effective worker is

$$\frac{Yt}{AN} = f\left(\frac{Kt}{AN}\right)$$

The graph of output and capital per effective worker looks the same as the previous one, only the axis have a different name, as you can see on slide 15.

An increase in A means that the labour needs less capital per effective worker. Being in the steady state means that the effective capital per worker is not changing anymore. Here, economic growth (or technological process) means that every worker is equipped with less labour.

Capital and labour
To determine output in the long run, 2 relations between output and capital should be noted:
- The amount of capital determines the amount of output being produced.
- The amount of output determines the amount of saving and, in turn, the amount of capital accumulated over time.

Understanding the steady state
To make it clear one more time we define the steady state in this and in the last lecture:
- In this lecture, the steady state is defined as the state of the economy in which the level of (HERE!: effective) capital per worker does not change over time.
- In last lecture, the steady state implied that the addition to the capital stock (investments) were exactly equal to the amount of capital that depreciates.

Next, we will see how much capital needs to be accumulated when we take into account technological progress and population growth.

When we want to stay in the steady state. What should happen to keep the amount of capital per effective worker constant when there is depreciation of capital?
Depreciation implies that there is less capital (per effective worker). Hence, to stay in the steady state there should be additional investment.

When we want to stay in the steady state. What should happen to keep the amount of capital per effective worker constant when there is population growth?
Population growth implies that there is still the same amount of capital. However, the amount of capital per worker is less. That means that the amount of capital per effective worker is less. Hence, to stay in the steady state, there should be additional investment.

When we want to stay in the steady state. What should happen to keep the amount of capital per effective worker constant when there is there is technological progress?
Technological progress implies that there is still the same amount of capital. There is also the same amount of capital per worker. Yet there is less capital per effective worker. Hence, to stay in the steady state, there should be additional investments.

Required investments in the steady state
The amount of capital that should be invested to keep the level of capital per effective worker constant is equal to

$$(\delta + g_A + g_N)\frac{K}{AN}$$

g is the growth rate. The graph on slide 23 is the same as before with the same dynamics, only different axis and the red curve contains different elements. Temporary growth is still obtained when there is growth in the capital stock.
The steady state is represented by K/AN. When A increases the whole ratio decreases. To keep the ratio constant, K must increase.
In the long run, capital per effective worker reaches a constant level, and so does output per effective worker. This implies that output (Y) is growing at the same rate as effective labour (AN). In order to keep the capital per effective worker constant when there is an increase in A, K must increase.

Technology in the steady state
In the steady state, output (Y) grows at the same rate as effective labour (AN); effective labour grows at a rate (g_A+g_N). Therefore, output growth in the steady state equals (g_A+g_N). The growth rate of output is independent of the saving rate. Because output, capital and effective labour all

grow at the same rate, (g_A+g_N), the steady state of the economy is also called a state of balanced growth.

If there is an increase in the savings, the green curve in the graph on slide 28 will shift upwards. The growth rate still is (g_A+g_N).

What we have seen now is that economic growth can be sustained in the long run due to technological progress.
The Solow model is an exogenous growth model because all variables that are responsible for economic growth are determined outside the model
The AK model is an endogenous model that can create growth under the assumption that technology is constant. Y = AK. There will be no returns to scale because the production function is linear.

$$\frac{Y}{N} = \frac{AK}{N}$$

$$\frac{Y}{N} = A(\frac{K}{N})$$

Required investment is

$$(\delta + g_N)\frac{K}{N}$$

In a graph you can see that if A is bigger than $\delta + g_N$ there will be transmission to the right and there will be no steady state. A will not increase but there still will be growth, which we call endogenous growth.
If A is smaller than $\delta + g_N$ there will be transmission to the left and the steady state will be the origin of the graph. This situation is also called the poverty trap. This occurs when the population grows but there is no technological process.