Chapter 3: Time value of money: An introduction

Cost-benefit analysis

When evaluating a decision, the valuation of the incremental costs and benefits, associated with that decision, is important. Identifying the costs and benefits of a decision is the first step in decision making. If the value of the benefits exceeds the value of the costs, the value of the firm will increase and a decision can be regarded as good. If you would like to compare costs and benefits that occur at different points in time, you have to put all costs and benefits in common terms. For this reason, we convert costs and benefits into cash today. If it's possible to express costs and benefits in terms of "cash today", it's a straightforward process to compare them and to determine whether the financial decision will increase the value of the firm.

A *competitive market* is a market in which the good can be bought and sold at the same price. The price is given and determines the cash value of the good. Because the good can be bought and sold at the same price, the personal preference and the opinion of the buyer/seller do not matter. It's easy to determine the best decision for the firm, if we use market prices to evaluate the costs and benefits of a decision in terms of cash today.

Valuation principle

The valuation principle provides the basis for decision making. The valuation principle is: the competitive market price determines the value of a commodity or an asset to the firm or its investors. The benefits and costs of a decision should be evaluated using those market prices. A decision will increase the market value of the firm, when the value of benefits exceeds the value of costs.

The valuation principle relies on using a competitive market price to value a cost or benefit. We cannot have two different competitive market prices for the same good. This established the *Law of one price;* in competitive markets, securities with the same cash flows must have the same price. According to this law, arbitrage opportunities cannot exist. The law of one price implies the price of a security should equal the present value of the future cash flows obtained from owning that security.

Arbitrage is the practice of buying and selling equivalent goods to take advantage of a price difference. We call the situation in which it is possible to make a profit without taking any risk or making any investment an arbitrage opportunity.

Time value of money and interest rates

There is a difference in value between money today and money in the future. Today a euro is worth more than a euro in one year, in general. This difference is called the *time value of money;* this is the observation that two cash flows at two different points in time have different values.

It's possible to convert money today into money in the future by depositing money into a savings account. It's also possible to convert money in the future for money today: borrowing money from the bank. The *current interest rate* determines the rate at which we can exchange money today for money in the future (like an exchange rate across time).

The *interest rate* is defined as the rate at which money can be borrowed or lent over a given period. The *interest rate factor* is the rate of exchange between dollars today and dollars in the future: *(1 + r).* This factor has units of "€ in one year/€ today". For example, when the interest rate is 4%, the interest rate factor is 1.04.

Example
Suppose a firm has got an investment opportunity. The cost will be €100.000, today. The benefit will be €102.000, in one year. Because of the time value of money, the cost and benefits aren't directly comparable (the net value isn't €2.000). To calculate the net value, we need to know the cost of the investment in one year. We can calculate the cost in one year by multiplying the cost today by the interest rate factor. The cost in one year will be €104.000 (if the interest rate is 4%). This means: the firm gives up the €104.000 it would have had in one year if it had left the money in the bank. Or, if the firm borrows the €100.000 from the same bank, the firm has got a debt of €104.000 in one year. When we have converted the costs and benefits in "dollars in one year", we can use the Valuation Principle to compare them and the compute the net value. The cost was €104.000 in one year, the benefit was €102.000, so the net value of this investment will be -€2.000 in one year. Because the net value is negative, the firm would reject the investment.

In the example we calculated the net value by computing the "cost in one year". In the same example, it's also possible to calculate the net value by computing the "benefit today". You can compute the "benefit today" by dividing the "benefit in one year" by the interest rate factor.

The *present value* (PV) is the value of a cost or benefit computed in terms of cash today. The *future value* is the value of a cash flow that is moved forward in time.

The *discount factor* is the value today of a dollar received in the future. This factor provides the discount at which we can purchase money in the future. The appropriate rate to discount a stream of cash flows to determine their value at an earlier time is called the *discount rate = r.*

To visualize the benefits and costs in time we often use a *time line*; a linear representation of the timing of (potential) cash flows. To differentiate between inflows and outflows on a timeline, inflows are positive cash flows (cash flows received) and outflows are negative cash flows (cash flows paid out).

Valuing cash flows at different points in time

There are three important rules central to financial decision making that allows us to compare or combine values across time.

1. Rule 1: Comparing and combining values

The first rule is that it is only possible to compare or combine values at the same point in time. Rule 2 and 3 show how to move cash flows on the timeline.

2. Rule 2: Compounding

The second rule states that to calculate a cash flow's value, you must compound it. *Compounding* is computing the return on an investment over a long horizon by multiplying the return factors associated with each intervening period. The *compounding interest* is the effect of earning 'interest on interest'.

Future value of a cash flow: $FV_n = C \times (1+r)^n$

3. Rule 3: Discounting

The third rule states that to calculate the value of a future cash flow at an earlier point in time, we must discount it. Discounting is finding the equivalent value today of a future cash flow by multiplying by a discount factor, or equivalently, dividing by 1 plus the discount rate. Present value of a cash flow: $PV = C / (1+r)^n$