Hoofdstuk 7

Bijlage 7.1

Requirements for a Distribution of a Discrete Random Variable

1. $0 \leq P(x) \leq 1$ for all x
2. $\sum_{\text{all } x} P(x) = 1$

where the random variable can assume values x and $P(x)$ is the probability that the random variable is equal to x.

Bijlage 7.2

Population Mean

$$E(X) = \mu = \sum_{\text{all } x} xP(x)$$

Bijlage 7.3

Population Variance

$$V(X) = \sigma^2 = \sum_{\text{all } x} (x - \mu)^2 P(x)$$

Bijlage 7.4

Shortcut Calculation for Population Variance

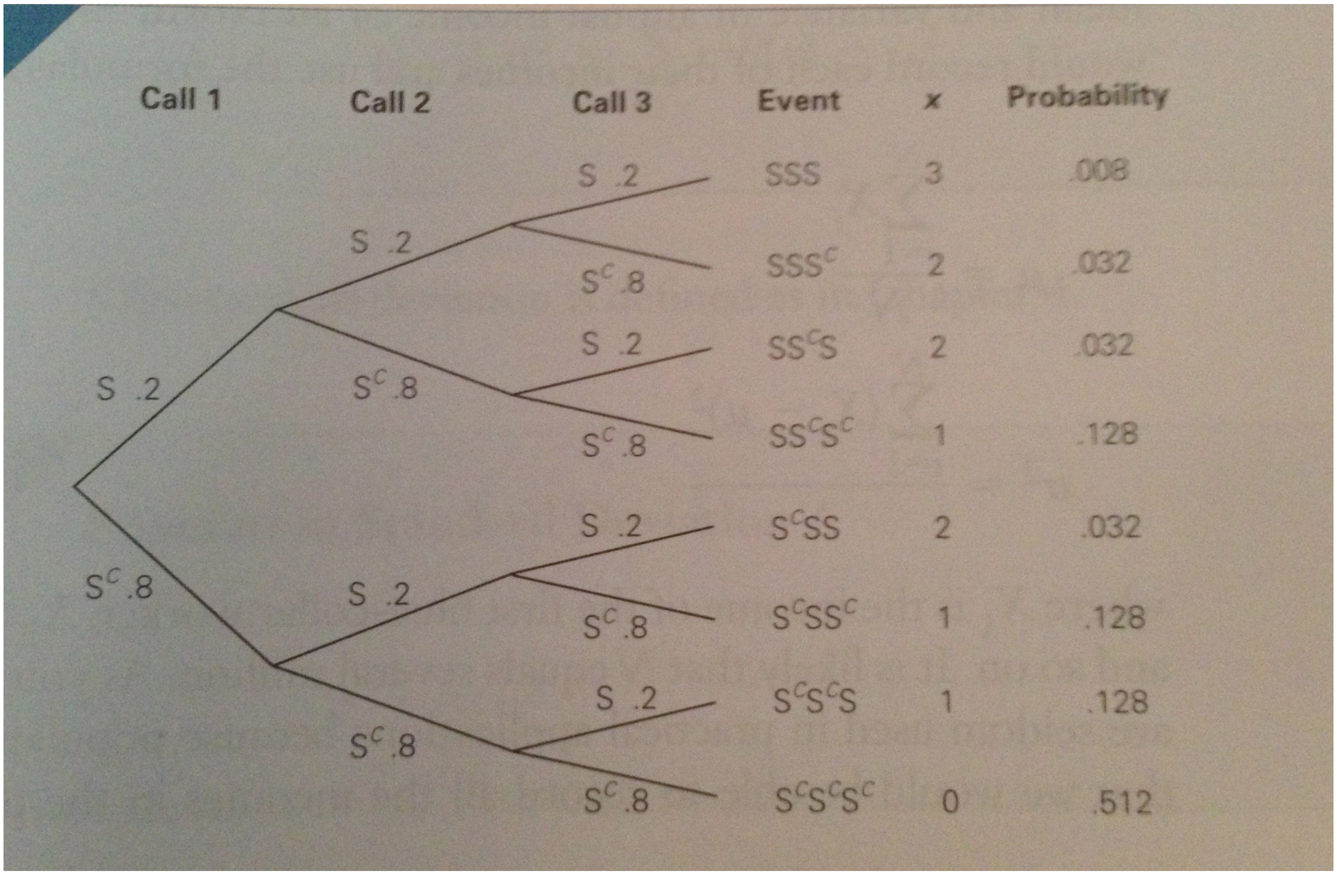
$$V(X) = \sigma^2 = \sum_{\text{all } x} x^2 P(x) - \mu^2$$

Bijlage 7.5

Population Standard Deviation

$$\sigma = \sqrt{\sigma^2}$$

Bijlage 7.6



Bijlage 7.7

Laws of Expected Value

1. $E(c) = c$
2. $E(X + c) = E(X) + c$
3. $E(cX) = cE(X)$

Bijlage 7.8

Laws of Variance

1. $V(c) = 0$
2. $V(X + c) = V(X)$
3. $V(cX) = c^2 V(X)$

Bijlage 7.9

Covariance

The covariance of two discrete variables is defined as

$$COV(X,Y) = \sigma_{xy} = \sum_{\text{all } x} \sum_{\text{all } y} (x - \mu_X)(y - \mu_Y)P(x, y)$$

Bijlage 7.10

Shortcut Calculation for Covariance

$$COV(X,Y) = \sigma_{xy} = \sum_{\text{all } x} \sum_{\text{all } y} xyP(x, y) - \mu_X \mu_Y$$

Bijlage 7.11

Coefficient of Correlation

$$\rho = \frac{\sigma_{xy}}{\sigma_x \sigma_y}$$

Bijlage 7.12

Mean and Variance of a Portfolio of Two Stocks

$$E(R_p) = w_1 E(R_1) + w_2 E(R_2)$$

$$V(R_p) = w_1^2 V(R_1) + w_2^2 V(R_2) + 2w_1 w_2 \mathrm{COV}(R_1, R_2)$$

$$= w_1^2 \sigma_1^2 + w_2^2 \sigma_2^2 + 2w_1 w_2 \rho \sigma_1 \sigma_2$$

where w_1 and w_2 are the proportions or weights of investments 1 and 2, $E(R_1)$ and $E(R_2)$ are their expected values, σ_1 and σ_2 are their standard deviations, $\mathrm{COV}(R_1,R_2)$ is the covariance, and ρ is the coefficient of correlation.

(Recall that $\rho = \frac{\mathrm{COV}(R_1,R_2)}{\sigma_1 \sigma_2}$, which means that $\mathrm{COV}(R_1,R_2) = \rho\sigma_1 \sigma_2$.)

Bijlage 7.13

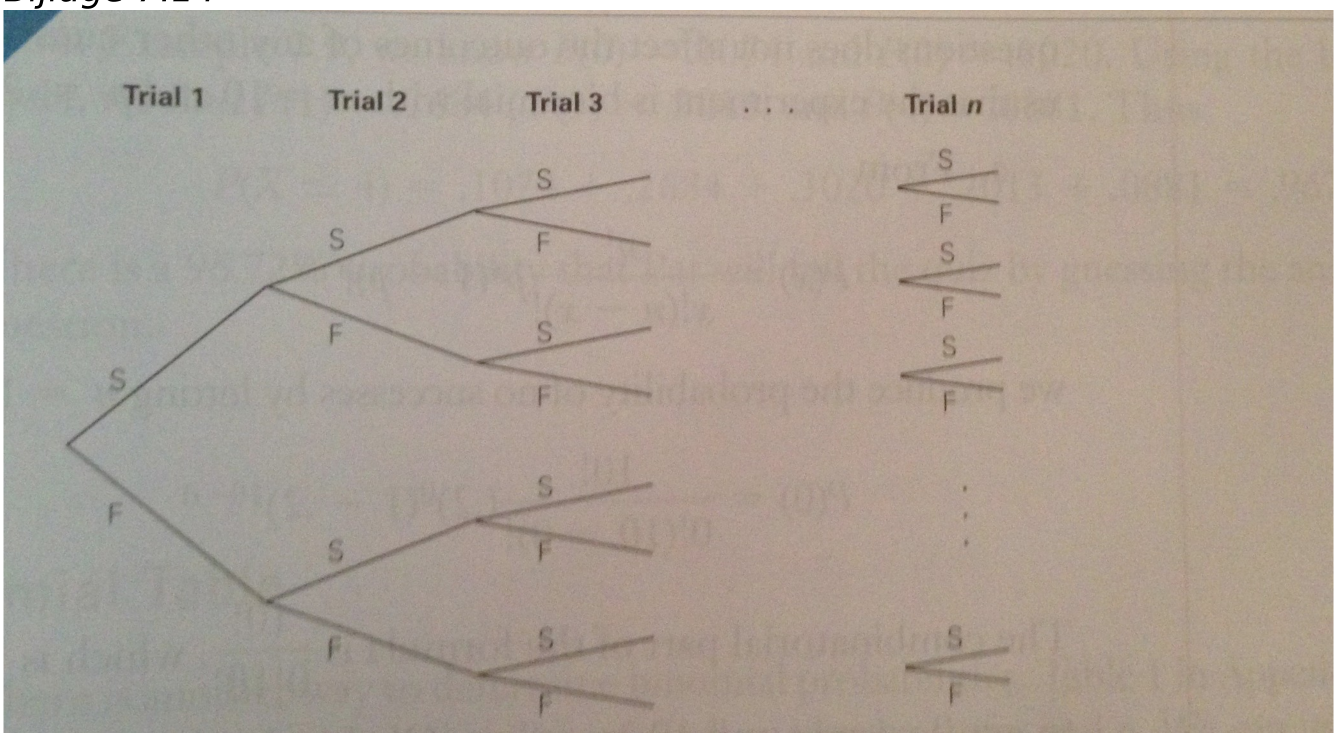
Mean and Variance of a Portfolio of k Stocks

$$E(R_p) = \sum_{i=1}^{k} w_i E(R_i)$$

$$V(R_p) = \sum_{i=1}^{k} w_i^2 \sigma_i^2 + 2 \sum_{i=1}^{k} \sum_{j=i+1}^{k} w_i w_j \text{COV}(R_i, R_j)$$

Where R_i is the return of the ith stock, w_i is the proportion of the portfolio invested in stock i, and k is the number of stocks in the portfolio.

Bijlage 7.14



Bijlage 7.15

Binomial Probability Distribution

The probability of x successes in a binomial experiment with n trials and probability of success $= p$ is

$$P(x) = \frac{n!}{x!(n-x)!}p^x(1-p)^{n-x} \quad \text{for } x = 0, 1, 2, \ldots, n$$